

Young Filipino Students Making Sense of Arithmetic Word Problems in English

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Young Filipino children are expected to solve mathematical word problems in English, a task which they typically encounter only in schools. In this exploratory study, task-based interviews were conducted with seven Filipino children from a public school. The children were asked to read and solve addition and subtraction word problems in English or Filipino. Analysis focused on how language influences problem solution, and on the reading and mathematical strategies used. Results showed that children (a) were better able to comprehend and solve problems given in Filipino, (b) rarely used reading strategies, (c) were not familiar with using drawings or objects to represent word problems, and (d) employed unitary counting procedures for problems involving multi-digit numbers.

Key words: Word problems; English language learners; Reading comprehension

Introduction

The 1974 Bilingual Education Policy in the Philippines mandated the use of English as the medium of instruction in mathematics. Mathematics textbooks and materials are written in English for all grade levels. Thus, Filipino children are expected to solve arithmetic word problems in English even from their first year of schooling. This expectation poses a very challenging task, particularly because children from non-affluent families typically

encounter English only in school and may not grow up proficient in English (Gonzales, 1989; National Statistical Coordination Board [NSCB], 2007; Young, 2002).

Although supported by scant literature, Filipino children appear to find mathematical word problems difficult (Bernardo, 1999; Brawner et al., 1999). These difficulties are frequently attributed to their inability to make sense of the problem due to lack of English language proficiency. Some studies suggest that the use of Filipino facilitates word problem solution (e.g. Bernardo, 1999). The Trends in International Mathematics and Science Studies (TIMSS) National Research Coordination Office (cited by Carteciano, 2005) asserts that language barriers and comprehension problems were among the “what-else-is-new problems” facing Filipino students who are learning mathematics in a second language.

There is extensive literature on how young children solve addition and subtraction word problems (for reviews, see Fuson, 1992b; Verschaffel, Greer, & De Corte, 2007). A well-established theoretical base has informed the design and implementation of successful professional development programs aimed to apply research to classroom instruction. However, there are no such initiatives specifically focused on helping Filipino children solve arithmetic word problems. There is a pressing need for intervention since an understanding of addition and subtraction is a foundation for later mathematics achievement (Irwin & Irwin, 2005; Pearn & Stephens, 2007).

We are currently planning a project aimed at designing an intervention program focused on addition and subtraction word problems for public school children in Metropolitan Manila. Given the limited research in this area, it is important to understand children’s strategies in solving word problems and factors that influence success. To this end, the first author administered a written test and conducted student interviews among young Filipino students. This paper reports on the student interviews, while the findings from the written test are reported elsewhere (Bautista, Mitchelmore, & Mulligan, 2009).

Background and Theoretical Perspective

The extensive research into children’s thinking in addition and subtraction situations has led to an understanding of how addition and subtraction concepts develop. Solving word problems competently involves more than looking for key terms (such as *altogether* or *more*) and mindlessly performing

an arithmetic operation on the given numbers in the problem. Rather, problem solvers need to attend to the “structure” of the word problem, or the relationships between the given quantities. Based on the word problem structure, children must then (i) construct an internal representation of the problem, and (ii) select a solution strategy (De Corte & Verschaffel, 1991; Mayer, 2003). We base our analysis on these two phases of word problem solving.

First, in constructing an accurate representation, the child has to read and make sense of the text. Reading is not the same as decoding, or the ability to recognise words (Stothard & Hulme, 1996). Instead, reading is the process of constructing meaning from the text; it is active and strategic (Eunice Kennedy Shriver National Institute for Child Health and Human Development [NICHD], 2000). Reading strategies such as thinking aloud, visualising, and connecting text to prior knowledge have been found to be effective for students when solving arithmetic word problems (Fogelberg et al., 2008).

Second, a solution strategy is selected based on the internal representation constructed from the child’s interpretation of the problem text. Although children’s internal representations are not observable, they can be inferred on the basis of their external representations such as the strategies they use to solve problems (Goldin & Shteingold, 2001). In the present study, solution strategies were analysed according to their level of abstraction (Carpenter & Moser, 1984); drawn representations were analysed according to the type of component signs present (Thomas, Mulligan, & Goldin, 2002); and strategies for tasks involving multidigit numbers were analysed according to children’s multidigit conceptions (Fuson et al., 1997).

Since Filipino children solve word problems in a second language, our analysis will also draw attention to how the language of the problem influences problem solution. There is extensive literature on the influence of the language of the problem on the solution performance of students learning mathematics in a second language (e.g. Abedi, Courtney, Leon, Kao, & Azzam, 2006; Clarkson, 2007; Dale & Cuevas, 1987; Secada, 1991). Research indicates that there is improved student performance when word problems are given in the students’ first language (Adetula, 1990; Bernardo, 1999). However, children’s failure to solve word problems cannot be wholly attributed to the inability to comprehend what was read. In our own study of 75 Filipino schoolchildren (Bautista et al., 2009), it was found that

presenting problems in Filipino improves, but does not guarantee, solution accuracy.

With the above theoretical perspectives, we aim to investigate how young Filipino public school children solve word problems. Most empirical data on Filipino children's performance on word problem solving were obtained from large-scale written assessments (e.g. Brawner et al., 1999). We supplement findings from written tests with an exploratory study utilising individual task-based interviews typical of cognitive-based studies (Goldin, 2000). Two key research questions are investigated:

1. What problem-solving strategies do Filipino children demonstrate when solving addition and subtraction word problems in English or Filipino?
2. How do children respond when encouraged to use different modes of representation?

In common with other single or multi-case studies (Yin, 2003), the aim of this study is not to make generalised conclusions but to explore students' activity and expose problem-solving strategies. The interviews reported here, together with the results of the written test (Bautista et al., 2009), will provide a more in-depth analysis of the differences between solving English and Filipino word problems.

Method

Participants

The participants were seven students studying in a public school in one of the most densely populated and poverty-stricken areas of metropolitan Manila. Malnutrition is a persistent problem for the students. Resources are inadequate, and it is common for classes to consist of 70 students. All participants were part of a community-based organisation which provides informal educational assistance to school-age members. The participants were 3 boys and 4 girls, ranging from Grades 1 to 4. Participant details are provided in Table 1. Pseudonyms are used for all participants. The interviewer is a long-time volunteer in the community, and the participants were already familiar with her.

At the time of the interviews, the participants had received instruction in addition and subtraction of whole numbers. The main difference between grade levels with respect to these operations is in the magnitude of the

numbers used, as determined by the Philippine Basic Education Curriculum (Department of Education Bureau of Elementary Education, 2003). Word problems were also included in the curriculum for the purpose of applying computational skills.

Table 1
Participant Details at the Time of the Interview

Name	Sex	Age (years/months)	Grade level
Mila	Female	6/1	1
Ramil	Male	7/1	1
Alma	Female	7/4	2
Teresa	Female	9/0	2
Vivian	Female	9/3	3
Jake	Male	10/0	4
Noel	Male	10/5	4

Although the curriculum documents include word problems as an application of an arithmetic operation, research shows that word problems may be solved using strategies that closely model the problem situation (Carpenter & Moser, 1984; De Corte & Verschaffel, 1987). Thus, the purpose of our interviews was not to determine whether children “applied the correct operation” but to assess their understanding of additive structures on the basis of their strategies.

Instruments

A set of 13 tasks was devised for the interview. Selected tasks are shown in Table 2. Eight of these tasks were constructed to represent the semantic categories of Riley, Greeno, and Heller (1983): Change problems 1-6; Combine problems 1 and 2; and Compare problems 1-6. The eight tasks from Riley et al’s categories involved small numbers—all sums were less than 12. Two non-routine tasks which are not solved by applying one arithmetic operation were included to elicit drawn or concrete representations (Kamii, Rummelsburg, & Kari, 2005). Two more tasks involved multidigit numbers. Finally, one Retelling task was included to reveal how the problem text was interpreted and how the problem representation was constructed (Verschaffel, 1994).

An English and Filipino version for each task was created, and back-translated to check for consistency.

Table 2
Selected Interview Tasks

Task	Problem
Retelling task (Compare 3)	Kiko has 7 books. Kikay has 3 more books than Kiko. [<i>May 7 libro si Kiko. Mas marami ng 3 ang libro ni Kikay kaysa kay Kiko.</i>]Please retell the problem to me. How many books does Kikay have?
Change 3 (from Riley, Greeno, & Heller)	Rico had 3 rice cakes. Tina gave Rico some more rice cakes. Now, Rico has 8 rice cakes. How many rice cakes did Tina give Rico? [<i>May 3 puto si Rico. Binigyan pa siya ni Tina ng mga puto. Ngayon, may 8 puto na si Rico. Ilang puto ang binigay sa kanya ni Tina?</i>]
Compare 1 (from Riley, Greeno, & Heller)	Nato and Tino went with their father to catch some crabs. Nato caught 8 crabs and Tino caught 5 crabs. How many crabs did Nato catch more than Tino? [<i>Sumama sina Nato at Tino sa kanilang ama para manghuli ng alimango. Nakahuli ng 8 alimango si Nato at nakahuli ng 5 alimango si Tino. Ilang higit na alimango ang nahuli ni Nato sa nahuli ni Tino?</i>]
Equalise 1 (from Riley, Greeno, & Heller)	Ruben picked 3 guavas. David picked 8 guavas. How many more guavas does Ruben have to pick for him to have equal number of guavas with David? [<i>Nakapitas si Ruben ng 3 bayabas. Nakapitas si David ng 8 bayabas. Ilan pang bayabas ang kailangang pitasin ni Ruben para magkaroon sila ni David ng parehong dami ng bayabas?</i>]
Drawing representation task (from Kamii, Rummelsburg & Kari)	People formed a line to buy lunch. I was standing in line and counted 3 people in front of me and 6 people behind me. How many people were in line altogether at that time? [<i>Pumila ang mga tao para bumili ng pananghalian. Habang ako'y nasa pila, nabilang ko na may 3 tao sa aking harapan at 6 sa aking likuran. Ilan lahat kaming nakapila?</i>]
Concrete representation task (from Kamii, Rummelsburg & Kari)	I have 12 pieces of candy. If I gave 2 pieces to my mother, 2 pieces to my father, and 2 pieces to my sister, how many pieces would I have left? [<i>Mayroon akong 12 na kendi. Kung magbibigay ako ng 2 kendi kay nanay, 2 kendi kay tatay, at 2 kendi kay ate, ilang kendi ang matitira sa akin?</i>]

Procedures

Since all participants were members of a community-based organisation which provides some form of support for its members, it was appropriate to conduct the interviews in the organisation's centre. All interviews were conducted by the first author. Problems were written on cards and presented to the child one at a time. Since reading strategies were included in our analysis, the participants in this study were expected to read the problems on their own. Red and green cubes, unifix cubes, counters, paper, and pencils were available. The interviewer explained to the child that they could use any of these materials if they thought it would help them reach a solution. The child was also asked to explain his or her solutions.

If a child gave an incorrect response, the interviewer provided a Filipino translation of the problem, or suggested the use of blocks, or pen and paper. Whenever possible, children were given as many attempts as they needed to solve the problem, and children moved on to a new task only after having successfully solved the previous one. The assistance provided by the interviewer was intended to determine potential approaches which could facilitate problem solution and inform the design of a subsequent intervention.

In some instances where a child failed to solve a problem, a task (cf. Wright, Stanger, Stafford, & Martland, 2006) involving counters corresponding to the problem's mathematical structure was given. For example, the corresponding task for the Change 3 problem (see Table 2) involved displaying 3 counters, and joining 5 more counters without allowing the child to see how many were added. The child was then informed (in Filipino) that there were now eight counters, and was asked how many counters were added.

During the course of the interviews, some children gave no response after they have read the problems written in English, and some lacked vocabulary knowledge in the English version of the problem. In these cases, they were not asked to attempt a solution and were given the Filipino version instead. Only Filipino versions of the problem were given for all succeeding tasks. All interviews were completed in less than 35 minutes. The entire interview was either audio- or video-recorded, and completely transcribed afterwards.

Results and Analysis

Children's solutions to each problem task were analysed with respect to the language of the problem, reading strategies, and mathematical strategies. We provide summary descriptions followed by a selection of some protocols to illustrate the results.

Language Effects

The results demonstrate that the language of the problem led to differences in children's solutions. First, there was a higher occurrence of non-attempts and errors when problems were given in English. Some problems not solved in English were solved in Filipino. Second, results show difficulties in solving Compare problems. A related finding was that children found Compare 1 more difficult than the Equalise 1 problem, even though both required finding the difference between two disjoint sets. Third, for five of the seven children, only Filipino versions of the problems were presented after one or two tasks because the use of English clearly prevented them from proceeding further along a solution. They either gave no response or did not understand some words in the text.

In Protocol 1, Teresa (Grade 2) gave no response when presented with the English Change 3 problem, "Jose caught eight fishes in the river. Jose gave Lourdes five fishes. How many fishes does Jose have now?" However, she gave a correct solution after she read the Filipino version.

Protocol 1

- Teresa: (starts reading the given problem) Jose (stops on 'caught').
- Interviewer: Caught.
- Teresa: Caught – caught eight fishes in the river. Jose gave Lourdes five fishes. How many fishes does – does Jose have now?
- Interviewer: *Naintindihan mo ba* [Do you understand]? *Yung – alam mo ibig sabihin ng caught* [Do you know what *caught* means]? (child shakes head) *Nakahuli* [caught]. River, *alam mo yung* [do you know] river? (child nods) *O sigge* [Alright], Jose caught –
- Together: Jose caught eight fishes in the river.

- Teresa: Jose gave Lourdes five fishes. How many fishes does Jose have now?
- Interviewer: *Kaya mo ba* [Can you do it]? (child nods) *Naintindihan mo na* [Do you understand]? (child nods) *O ilan* [How many]? (9 seconds pause) *Subukan natin Tagalog ha* [Let us try the Filipino version]. *Ito* [Here].
- Teresa: [reads the given problem] *Nakahuli ng walong isda si Jose sa ilog* [Jose caught eight fishes in the river]. *Binigyan ni Jose si Lourdes ng lima – limang isda* [Jose gave Lourdes five – five fishes]. *Ilang isda ang natira kay Jose* [How many fishes does Jose have now]? (one second pause) *Tatlo* [Three].
- Interviewer: *O, tatlo* [Three]. *Ang bilis a* [That was fast]. *Pa'no mo nakuha yung tatlo* [How did you get three]?
- Teresa: *Nag-minus* [Used minus].

Teresa's poor English language knowledge prevented her from solving the problem. However, when the problem was presented in Filipino, she was able to solve the problem without having to resort to using fingers or concrete objects. She immediately recognised that the problem represented a subtraction situation, and accordingly used her knowledge of subtraction facts to solve the problem.

Protocol 1 also suggests that children's limited English vocabulary may explain why there are several non-attempts for English problems. Another possible explanation is that children fail to relate particular words to the problem situation. As Protocol 2 shows, Noel (Grade 4) knew the meaning of the word *change* but in an inappropriate context.

Protocol 2

- Noel: Arman had one hundred pesos in his pocket. He bought a pizza for eighty-four pesos. How much is his change? One hundred eighty-four. (indistinct speech) *Mahirap* [Difficult].
- Interviewer: *Mahirap ba* [Was it difficult]? *Pero naintindihan mo yung tanong* [But do you understand the question]? *Alam mo ibig sabihin ng change* [Do you know what *change* means]?
- Noel: *Opo* [Yes].

- Interviewer: *Ano* [What]?
- Noel: *Papalit* [Swap].
- Interviewer: *Hindi, ano yan* – sukli [No, that's the balance of money returned].
- Noel: *Ay, sukli* [Ah, balance of money returned].
- Interviewer: Ok.
- Noel: *Ah, alam ko na* [Now I know]! [...] Sixteen *sukli niya* [was his change].
- Interviewer: *Sa'n mo naman nakuha ang sixteen* [Where did you get sixteen]?
- Noel: *Kasi yung eight, dinagdagan ko ng sampu* [Because I added ten to eight]. *Tapos yung four, dinagdagan ko ng anim* [Then I added six to four]. *Para maging* [to make] one hundred.

In the above protocol, Noel initially gave a quick answer but appeared unsure whether it was correct. However, when he was told what *change* meant in the problem context, he understood what to do. This protocol highlights another obstacle which Filipino students face. They are exposed to English primarily in school (Young, 2002). This situation deprives them of a rich opportunity for English vocabulary acquisition. Reading experts (e.g. see Armbruster, Lehr, & Osborn, 2001 for reviews) generally agree that children learn vocabulary when they hear and see words in multiple contexts, especially words with multiple meanings (Nelson & Stage, 2007). As a result, the limited exposure of Filipino children to the English language impedes their word problem solution.

The interviews also showed that there was general difficulty in solving Compare problems in both languages. For example, in the Retelling task, the children either failed to comprehend the problem, and those who did required many attempts. The only exception was Jake who managed to comprehend the Filipino version of the Recall task and solve the Filipino version of the Compare 1 problem. For the rest of the participants, Compare problems were difficult even when translated to Filipino. This finding is consistent with the results we obtained from our written test (Bautista et al., 2009).

The most common form of miscomprehension was interpreting a relative quantity to be an absolute quantity. This happens when children miscomprehend or ignore the comparative term and assign the relative quantity to one person (see Protocol 3). Here, Jake (Grade 4) recalled an English Compare 5 statement in the Retelling task.

Protocol 3

Jake: (reads the text) The king has 7 gold coins. The queen has 3 more gold coins that, than the king.

Interviewer: Ok. *Kaya mo bang sabihin sa'kin kung ano ang nakasulat dito* [Can you tell me what is written here]? *Kunwari siya, hindi siya marunong magbasa* [For example, he cannot read] (pointing to a doll). *Kaya mo bang ikwento sa kanya kung anong nakasulat diyari* [Can you tell him (the doll) what is written there]?

Jake: *Opo* [Yes].

Interviewer: *O anong nakasulat dito* [What is written here]?

Jake: *Ang hari ay mayroong pitong, pitong gintong* [The king has seven, seven gold] –

Interviewer: Ok.

Jake: Coins.

Interviewer: Ok.

Jake: *At ang reyna ay may tatlong* [And the queen has three] gold – *gintong* [gold] coins *mas* [more] than the king.

Interviewer: Ok. *E kung tinanong ko sa 'yo, ilang* [If I asked you how many] gold coins *meron yung hari* [does the king have]?

Jake: Seven.

Interviewer: *E yung reyna* [And the queen]?

Jake: Three.

Interviewer: Three. How many gold coins...

Jake: Does the queen have?

Interviewer: *Ang sagot mo* [Your answer is]?

Jake: Three.

In the above protocol, observe that Jake chose to verbally retell the problem in Filipino. He did not have any problem translating the first statement. However, when he tried to retell the second statement, he was only able to translate the phrase, "The queen has three gold," and followed this with the Filipino-English phrase "coins *mas* than the king." Further, the word *mas* [more] is not normally said after a noun, and this makes the translation senseless. He was employing a literal translation, translating words one at a time, and failed to relate the comparative term *more than* to its referents. The result was a nonsensical translation which did not help him understand the text and solve the problem. His final solution was in fact the correct solution to his own interpretation of the problem. He assigned one quantity (7) to the king, and another quantity (3) to the queen. The difficulties may be attributed to language because he was able to solve both the Compare Retelling task and the Compare 1 problem when these were presented in Filipino, but not when these were presented in English.

The only other child who successfully solved a Compare problem (in Filipino) was Teresa. The rest of the children, however, had difficulties. Levi misinterpreted the difference as an assigned quantity. Alma added the two numbers in the problem. Vivian gave no response. These difficulties were not found with the similar Equalise 1 problem. These three children successfully solved the problem, and two of them solved it in their first attempt. The difficulties associated with Compare problems have been found in monolingual students as well. Like the participants in our study, monolingual students perform better when solving Equalise problems than Compare problems (De Corte, Verschaffel, & De Win, 1985). Several researchers (Cummins, 1991; Fan, Mueller, & Marini, 1994) agree that these two problems require the solver to find the difference between two disjoint sets, and that these differ only in the problem text. Cummins (1991) argues that the Compare problems are particularly difficult because children may not have the linguistic knowledge to comprehend comparative language.

Finally, the children's unfamiliarity with the English language is another important finding. It is well-acknowledged that children encounter English only in schools (Young, 2002), and national achievement tests confirm that majority of the students have not mastered English (NSCB, 2007). Our study shows similar results. When problems were presented in English, all participants, except for Jake and Noel (Grade 4), did not attempt to read the text. They may have been overwhelmed by the text, or lacked vocabulary

knowledge of common English words. For example, some specific words which were not understood were “said,” “arriving,” and “caught.” Not only did they have problems with mathematical terms, but also with common English words. For these children, the advantages of being bilingual (e.g. Clarkson, 2007) were not realised because they were not proficient in one language. It was thus necessary to present all succeeding problems in Filipino, so that the children had a better chance of understanding the problem’s mathematical structure.

Reading strategies

In this section, we highlight three findings. First, some children had difficulties in decoding English text. This was especially true for the younger participants. Second, the ability to decode text did not guarantee text comprehension, even when problems were given in Filipino. In some instances, children who could not solve a word problem were able to produce a correct solution when the task was presented using counters. Third, reading strategies were rarely used. We now present examples of these general findings.

For some children, it was more difficult to decode English text than it was to decode Filipino text. Protocol 4 shows the difficulties which Ramil (Grade 1) encountered when decoding the English Change 1 problem: “There were 6 people who came to my place for dinner. Seven more people rang up and said they were coming. How many people am I having for dinner?”

Protocol 4

- Ramil: [reads the given problem] There were six—*ano ‘to* [what is this]?
- Interviewer: People.
- Ramil: People. (silence)
- Interviewer: Who.
- Ramil: Who. (silence)
- Interviewer: Came.
- Ramil: Came to my ha—*ano ‘to* [what is this]?
- Interviewer: Place.

- Ramil: Place for din – ner.
Interviewer: Dinner.
Ramil: Dinner. Se – six?
Interviewer: Seven.
Ramil: Seven ma.
Interviewer: More.
Ramil: Pee –
Interviewer: People.
Ramil: People ra-rang up and sa –
Interviewer: Said.
Ramil: Said they were com –
Interviewer: Coming.
Ramil: Coming. Hu –
Interviewer: How.
Ramil: How many pee –
Interviewer: People.
Ramil: People on – am ee
Interviewer: I
Ramil: I ha-having for dinner?

By contrast, Protocol 5 shows how Ramil read the Filipino version of the same problem.

Protocol 5

- Ramil: (reading accurately, but with frequent pauses) *May six na taong dumalo sa bahay ko upang maghaponan. Pitong tao pa ang tumawang – tumawag sa telepono para sabihin – hing darating sila. Ilan lahat ang mga bisita na mag – magsasalo-salo sa hapunan?*

Protocols 4 and 5 highlight the difference in Ramil's ability to decode English and Filipino text. Based on the types of reading miscues identified in the recent Handbook of Reading Assessment (Bell & McCallum, 2008), the 15 miscues in reading the English problem were as follows: four helper-supplied words, three pauses, and eight substitutions. There were 13 miscues in reading the Filipino version: all consisting of pauses, except for one self-correction. Although there was almost the same number of miscues, the types of miscue were completely different. Observe, for example, that Ramil was able to decode the Filipino text without any assistance from the interviewer.

These episodes also suggest one possible reason why Filipino students find it difficult to decode English text. Ramil primarily decodes text one syllable at a time. This may be an adequate strategy when reading Filipino because spelling corresponds very closely to how words are spoken (Gonzales & Rafael, 1981). But English text can only be decoded properly when "the target word is in the learner's oral vocabulary" (NICHD, 2000, p. 4-3). Since the children in this study are not familiar with many English words, they apply Filipino rules for decoding English text. An example of this is found in Protocol 4. If the word *said* is pronounced the way it was written, it would be read as *sah-eed*—which was how Ramil started to decode this word. Students with decoding difficulties may likewise find word problems difficult because they read "so slowly and laboriously that before they come to the words at the end of the sentence, they have forgotten those at the beginning" (Helwig, Rozek-Tedesco, Tindal, Heath, & Almond, 1999, p. 114).

Presenting the problem in English not only impeded decoding ability but also text comprehension. Protocol 6 shows how Vivian (Grade 3) failed to construct meaning from the Change 3 problem, "Rico had three rice cakes. Tina gave Rico some more rice cakes. Now Rico has eight rice cakes. How many rice cakes did Tina give Rico?"

Protocol 6

Vivian: *Sulat* [Write]?

Interviewer: *Hindi pwede mong* [No, you can] (indistinct word) *pwede mo nang* – *pwede mo nang subukang ano – sagutan* [you can already try to solve]. *Naintindihan mo ba* [do you understand]? (child nods) *Sige nga, ano – anong – Pwede ka*

mag-drawing, pwede mong gamitin 'to para tulungan kang sumagot [You can draw, you can use any of these to help you solve] (interviewer points to the blocks on the table).

- Vivian: *Anong gagawin ko* [What will I do]?
- Interviewer: *O, kagaya niyan* [Like this]. *Rico had three rice cakes.*
- Vivian: *Susulat ko dito* [Will I write here]?
- Interviewer: *Ikaw, kaya mo na bang sagutin mag-isa* [Up to you, can you already solve on your own]?
- Vivian: (shakes head)

In Protocol 6, Vivian did not know how to proceed. The child offered no response other than to ask whether she needed to write on the paper provided to her. In contrast, when the Filipino version of the problem was given, she was able to recognise that she was being asked how many rice cakes Tina gave Rico. Although Vivian ultimately failed to provide a correct solution, she manifested partial comprehension of the problem. Because she did not provide a correct solution, she was subsequently given a screened task involving counters (cf. Wright et al., 2006) which modelled the problem structure. Vivian was able to solve this task without any observable gesture. When asked how she solved the task, she reasoned that there must have been four additional counters because the initial five counters became nine.

Although Vivian had no difficulties in solving the screened task with the same mathematical structure as the initial problem, she still failed to solve the word problem when it was presented again in Filipino. She repeatedly mentioned that Tina gave Rico eight rice cakes, which is an incorrect interpretation of the text. Failure in comprehending the text is the most likely reason for her inability to solve the problem because she had the ability to solve a similar screened task when it was not embedded in text. Attempts to comprehend the problem text, even when it was read to her, were unsuccessful. The inability to retrieve information from written text was evident when she was asked for the number of rice cakes Rico has at present – she merely had to read the problem because this information was provided. However, she did not look for this information in the text. Instead, she added the two given numbers.

The example above is an illustration of a lack of interaction with the text, even when presented in Filipino. Protocol 70 provides a further illustration

of this lack of interaction. Here, Ramil (Grade 1) was solving the Filipino version of the problem “There are 6 visitors in my house. Seven more people said they were arriving. How many visitors will I have?”

Protocol 7

Ramil: (reading the problem in Filipino) *May anim na bisita sa aking bahay. Pito (...) tao pa ang nagsabing darating sila. Ilan lahat ang aking mga bisita?*

Interviewer: *Mm. Kaya [So]?*

Ramil: *Anim [Six].*

Interviewer: *Anim ang bisita [Six visitors].*

Ramil: *Bisita [Visitors].*

Interviewer: *Ngayon [Now].*

Ramil: *Ngayon [Now].*

Interviewer: *E ilan pa daw yung darating [How many more are coming]?*

Ramil: *Pito, pito [Seven, seven].*

Interviewer: *O, basahin mo ulit yung tanong [Read the question again].*

Ramil: *May anim [There are six]—*

Interviewer: *(...) yung huli, yung huli lang [Just the last part].*

There was no attempt to reread the problem, and an answer (six) was produced immediately. When he was asked to read the question so that he would know what to do with the information, he proceeded to read the text from the beginning. Reading, for him, meant decoding the text once again. There was no attempt to select relevant information or construct meaning from the text.

With the exception of Jake (Grade 4), children read the text only once for every problem (English or Filipino). They did not reread or refer back to the text, except when prompted by the interviewer. When they could not understand what they had read, they either gave no response, or they provided a guess. During the few times they asked questions, these were not meant to clarify meaning but to learn how to decode particular words, as shown in Protocol 4. The only participant who displayed reading strategies was Jake (Grade 4; see Protocol 8).

Protocol 8

- Interviewer: *Eto* [Here]. (interviewer shows the word problem)
- Jake: (child reads the problem) Ruben and David went to pick some ripe guavas. Ruben picked 3 guavas. David picked 8 guavas. How many more guavas does Ruben have to pick for him to have equal number of guavas with David? (indistinct word) David – *pumunta* [went], *kumuha* [got] *ng guavas si Ruben, kumuha ng tatlong* [three] *guavas, si David, walo* [eight]. *Ilang* [How many] – how many more – *ilang, ilan pa yung* guavas does Ruben mmm, *maku* – does Ruben have before (indistinct word) uhhh, Ruben got...*tatlo* [three] *nakuha ni Ruben*. David, eight. *Ilan pa daw* [how many] *kail...ni Ruben para ma-magkagaya sila ni* [same with] – equal – *ibig sabihin* [meaning], *magkaparehas* [same] *kay Ruben*. Five.

(To avoid confusion with Jake's own translation, we have translated only selected words in this protocol.)

Protocol 8 shows how Jake constructed meaning from the text. He paused regularly to construct meaning while he was reading the text. He thought aloud and paraphrased the text to facilitate understanding. He reread the text, and pointed to some parts of it to help organise the information. Even without being asked to, he translated the text line by line into Filipino in his effort to make sense of the problem. He did not devise a solution plan as soon as he decoded the text. Rather, he used various reading strategies which facilitated problem comprehension before deciding on a solution strategy. There was evidence of thinking during and after reading. Eventually, he was able to construct an appropriate problem representation and produce the correct solution.

Mathematical Strategies

The strategies used in the children's first solution attempt for each task were recorded in a matrix, and coded using categories developed by Carpenter and Moser (1984) and Fuson et al. (1997). Several common features of the children's preferred strategies were found. First, there were only two occasions where the counting sequence was used. Second, children did not know how the use of blocks or drawings could help them solve the problems.

Third, the strategies children used for multi-digit numbers were indicative of unitary conceptions. It was not uncommon for children to count objects by ones, even for two-digit numbers. Finally, children used their own strategies, and not those encouraged by classroom instruction. We now present protocols and children's work samples to describe these results in more detail.

Only Jake used the counting sequence to solve problems. But although he could do this efficiently for smaller numbers, he could not perform this strategy for larger numbers where he resorted to using concrete objects. The limited use of the counting sequence was also reported by De Corte and Verschaffel (1987), who felt the reason was that the curriculum "entirely disregarded or even discouraged [counting strategies]" (p. 370). The same situation applies in the Philippines – the curriculum jumps directly from the use of physical models to the application of arithmetic operations to solve word problems (Department of Education Bureau of Elementary Education, 2003). Fuson (1992a) argues that it is an erroneous oversimplification to assume that children can move directly from using concrete objects to using number facts.

The second finding was that the children preferred to work mentally, choosing not to use the blocks or drawings, even when these have been shown to help children organise their thinking and solve problems (Fan et al., 1994; Kamii, 2001; Outhred & Sardelich, 2005). Even when the use of blocks was suggested as a way to correct initial errors, some children did not know how the blocks could help them solve the problem, as exhibited in Protocol 9. This interaction occurred after the interviewer suggested the use of blocks following an incorrect response by Teresa (Grade 2).

Protocol 9

Teresa: *Si Rico ay may tatlong puto* [Rico had three rice cakes].

Interviewer: *Tatlong puto* [Three rice cakes]. *Sige nga, hanap ka diyan ng pwedeng pakita mo sa'kin ano yung tatlong puto* [Look for something there that can show me three rice cakes] (child gets three green blocks). Ok. *O, lagay mo diyan* [Place it there]. Ok. *Si Rico ay may tatlong puto. Binigyan pa siya ni Tina ng puto. Ngayon, ito yung puto n – nino* [Rico had three rice cakes. Tina gave him some more rice cakes. Now,

- whose are these rice cakes] (interviewer points to the three green blocks)?
- Teresa: Rico.
- Interviewer: Rico. *Ngayon, dahil binigyan siya ni Tina ng puto, mayroon na siyang* [Now, because Tina gave him more rice cakes, he now has]—
- Teresa: *Walo* [Eight].
- Interviewer: *Walong puto* [Eight rice cakes]. Ok. *Ngayon mayroon na siyang walong puto. Sige nga* [Now he has eight rice cakes. Go on] (child gets eight red blocks).

The above protocol shows that when children used blocks, they did not necessarily understand how they could use these to model the quantities and relations in the problem. Here, the interviewer read the problem line by line, allowing Teresa to model the action in the problem in segments. When the interviewer stated that there are now eight rice cakes, Teresa picked eight more blocks instead of adding more blocks to the original set of three blocks. It appears that Teresa used the blocks primarily to represent the two numbers in the problem, and not the relationship between these numbers. This may be interpreted as another form of “number grabbing” where the focus is directed only to the numbers and not to the problem structure.

As well as not using the blocks to represent the problem structure or its solution, using drawings also seemed to be a foreign strategy for the children. It was remarkable that children chose not to draw even in the Drawing Task where they were specifically instructed beforehand to draw. Despite being prompted, many were unsure of what to do.

As an example, consider the case of Ramil (Grade 1) who was prompted to draw when he initially failed to answer the Change 1 problem (refer to Protocol 7). His drawing (Figure 1) did not include any mathematical features, such as tally marks or symbols, and was quite idiosyncratic (Thomas et al., 2002). It appears that his attempt to draw was not focused on finding a solution to the problem. Much effort was placed on drawing superficial features and some additional details which were not given in the text, such as a house with a chimney and a person holding a mobile phone. This idiosyncratic drawing represented the situation, but it did not represent the problem’s mathematical structure. “Decorative pictures” such as this, do

not enhance understanding or facilitate problem solution, and may even lead to negative results (Berends & van Lieshout, 2009; Elia, Gagatsis, & Demetriou, 2007).



Figure 1. Ramil's drawn representation.

One possible explanation for why the children in this study failed to use blocks or drawings is that the curriculum (Department of Education Bureau of Elementary Education, 2003) emphasises symbolic representations of word problems. Verbal, concrete, or pictorial representations are largely ignored. Children's limited experience with various modes of representation could hinder the development of mathematical strategies for additive word problems because mathematical understanding relies heavily on the flexible use of multiple representations (Lesh, Post, & Behr, 1987; Maclellan, 2001).

The solution procedure recommended by the curriculum is also highly prescriptive. When children solve word problems, they are required to state what is asked and what is/are given, look for the word clues, select the correct arithmetic operation, and transform the word problem into a number sentence. These adult-imposed strategies impede and ignore children's own strategies and compel them to use strategies which may be meaningless to them.

The third finding from the analysis of mathematical strategies is that these strategies were indicative of a unitary conception of multi-digit numbers. For example, Ramil (Grade 1) and Jake (Grade 4) used blocks to solve the multi-digit problems. To find $100 - 84$, Ramil constructed a set of 100 blocks, set aside 84 blocks, and counted the remaining blocks to obtain

the answer. Jake employed a similar strategy when solving $42 + 36$. He first counted on by six to obtain 48. To proceed from 48, he counted 30 blocks and again counted on from 48 with the aid of the blocks. Both of them counted the blocks one by one, without reference to any grouping. This strategy is indicative of a unitary conceptual structure (Fuson et al., 1997) which leads to error-prone and time-consuming procedures. Indeed, in this study, Ramil miscounted the remaining number of blocks.

Multi-digit problems also prompted Teresa and Vivian to draw sticks or circles which they could count by ones. Figure 2 shows Vivian's solution to the Filipino version of the problem: "The teachers had a meeting. Out of 65 teachers, only 48 attended. How many did not attend?" She drew 65 sticks in rows of ten, and wrote the cumulative number of sticks at the end of each row. Next, she looked for the 48th stick by counting, "ten, twenty, thirty, forty" while she pointed at the numbers in synchrony. When she reached forty, she counted eight more sticks by ones to reach the 48th stick, and marked all the remaining sticks. Finally, she counted the marked sticks by ones to obtain the correct solution to the problem.

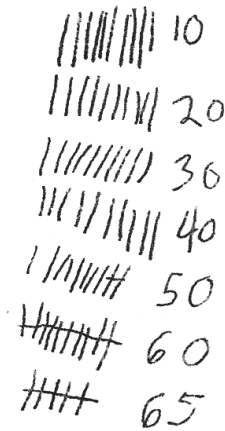


Figure 2. Vivian's solution strategy.

Vivian's solution showed evidence of grouping. Her skip count by tens was indicative of a sequence tens and ones conception. However, she was only able to use the groupings of ten to find the 48th stick, but not to solve 65 - 48. She did not realise that the 51st to 60th sticks formed a group of ten. Instead, she counted the sticks by ones. Thus, Vivian had constructed a partial sequence tens and ones conception.

Vivian's strategy also demonstrated a reliance on visible objects to count. She made sense of the count sequence, 10, 20, 30,.. by relating it to groups of ten sticks which she could perceive. Also, she did not use the economical counting on procedure for these subtraction tasks even when the two given numbers were relatively close to each other. This inefficient strategy suggests a lack of understanding of addition and subtraction as inverse operations.

Finally, episodes from the interviews demonstrate that the children relied on their own strategies rather than on those imposed by classroom instruction. No participant wrote down what was asked, looked for word clues, or wrote a number sentence, as recommended by the curriculum. One can thus question whether it is appropriate to include word problems only as an application of an arithmetic operation, and whether it is appropriate to prescribe a rigid list of steps to follow. Children who solved tasks involving multidigit numbers relied on unitary counting or mental strategies, and not on the standard algorithm taught in the classroom. It is likely that children fail to recognise how the mathematics being taught in class can "contribute to learning and problem solving" (Maclellan, 2001, p. 74).

Implications

Our analysis of children's strategies suggests that minimal interventions such as those used in this study (e.g. providing questioning strategies, suggesting the use of blocks or drawings) do not, by themselves, facilitate problem solution. Efforts need to be made to ensure comprehension of the language of the problem and to enrich the limited range of reading and mathematical strategies.

Our findings raise several considerations for designing an intervention program aimed to help students use efficient strategies to solve addition and subtraction word problems in English. First, because children had difficulties with common English words, vocabulary instruction and engaging children in English conversation must be an integral part of the

program. Although the children in this study were only asked whether they understood individual words in the problems, it is also possible that they may not understand phrases or semantics. To guide the design of an intervention, it would thus be worthwhile to ask children, in future interviews, whether they understand particular phrases or sentences in word problems.

Second, because decoding difficulties make word problem solving unnecessarily more arduous, the intervention must consider how children who still cannot decode text may benefit from mathematics instruction. They would probably learn best by working with oral word problems as a means to develop addition and subtraction concepts (cf. Zhou & Peverly, 2005).

Third, our findings indicate that children do not have a wide range of reading strategies for comprehending English or Filipino text. These reading strategies have to be developed. Modelling effective reading strategies has been shown to improve comprehension (Armbruster et al., 2001) and word problem solving (Foster, 2007). By having reading strategies modelled for them, children could learn how they themselves should interact with the text.

Fourth, the reliance on unitary counting and the absence of advanced strategies such as the use of derived facts highlight the need for developing numeracy. The intervention should provide emphasis on strategies based on the counting sequence (Fuson, 1992a) and on structuring numbers (Wright, Ellemor-Collins, & Lewis, 2007) because these lead to more sophisticated addition and subtraction strategies.

Finally, children's limited use of representations in problem solving reveals the need to provide them with diverse ways of representing word problems. Transforming word problems into symbolic form is not the only strategy to solve problems. Role playing or using drawings or concrete objects to model the action in the problem should be encouraged. Children should also be expected to communicate their strategies and justify their solutions. Further, there is a need to teach children to focus their representations on the problems' mathematical structure rather than on superficial features.

It is conjectured that by providing children with the representational and arithmetical tools outlined above, word problem solving performance would improve. Ultimately, the aim is to determine potential interventions

to help children make sense of word problems and to improve problem solving performance in the early grades, before linguistic and mathematical demands become more complex. An intervention program based on insights from this study is currently being trialled for small-group tutoring of young Filipino children.

Although this study has focused on improving the word problem solving performance of Filipino students, the results also have implications for any context where large numbers of students learn mathematics in a language they do not ordinarily encounter outside school, for example, in Malaysia where students or teachers are not fluent in the second language (Barwell, Barton, & Setati, 2007; Clarkson & Idris, 2006). The present research becomes even more valuable because most studies on mathematics for second-language learners have been focused on immigrant communities in developed countries.

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